GLOBAL MODELING OF MICROWAVE DEVICES USING WAVELETS

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ABSTRACT

A new approach to the simultaneous modeling of semiconductor devices and all the circuit elements is discussed in this paper. The electromagnetic wave propagation through semiconductor devices is modeled by using interpolating wavelets. The solution is developed in the time-domain. Examples of device and circuit simulations are presented.

INTRODUCTION

Highly packed modern integrated circuits consist of closely spaced active and passive devices, with many levels of transmission lines and discontinuities. The circuit performance may be adversely affected by the high density, due to unwanted effects such as crosstalk, caused by coupling, surface waves, and unintended radiation, to name just a few. Evidently, careful circuit designs must be developed based on advanced design tools that consider all the circuit elements simultaneously, including the active devices, the passive components, the radiation elements and the package. The possibility of achieving this global circuit modeling was demonstrated in [1]-[2].

Recently, a new category of orthogonal systems, "orthogonal wavelets", has appeared on the scene. The wavelet expansion has proven to be an efficient method in the approximation of functions. Different classes of wavelets have been used to analyze the passive circuit components. For example, Daubechies wavelets have been employed in [3] for the analysis of microstrip floating structures. In [4] the wavelet expansion method based on periodical wavelets in conjunction with the boundary element method has been applied for the analysis of multiconductor transmission lines.

To achieve the desired global modeling approach, we employed wavelet functions for modeling of active semiconductor devices. In this paper we consider the application of multiresolution analysis for semiconductor device modeling. The use of scaling functions and wavelets as a complete set of basis functions is called multiresolution analysis [5]. To derive a new algorithm, the potential distribution inside the semiconductor and the electron and hole current densities are represented by a two-fold expansion in scaling functions and wavelets. Using only scaling functions allows correct modeling of smoothly-varying electromagnetic fields and material parameters. In regions with strong field variations additional basis functions (wavelets) are introduced. In our derivations, we use a special class of wavelets, namely the interpolating wavelets. This wavelet system has already been applied to the solution of boundary problems for partial differential equations (PDE). For this type of wavelets, the evaluation of differential operators is simplified due to the simple representations in terms of cubic polynomial functions in the space domain.

Several different approaches for solving PDE using wavelets have been considered. Jameson [6] used wavelets for finding where to refine the grid in a finite difference method. It has been noticed by several authors that nonlinear operators such as multiplication are too computationally expensive when done directly in a wavelet basis. There were several attempts to deal with this problem. Keiser [7] has used Coifman wavelets to get approximations of point values in a wavelet method, thus simplifying the treatment of nonlinearities. In this paper we will follow the idea of Hölmstrom [8] to deal with nonlinearities using the so called Sparse Point Representation (SPR).

MODELING OF SEMICONDUCTOR DEVICES USING INTERPOLATING WAVELETS

The idea behind using a wavelet basis is that certain functions are well compressed in such a basis. It means that few basis functions are needed to represent a function with a small error. For example, consider a function which is represented by N points on a uniform grid, and the same function is represented, with the same error δ, by N_s wavelet coefficients, for $N_s \ll N$. We would like to be able to compute derivatives and multiply functions in this wavelets basis in $O(N_s)$ time. The interpolating wavelet transform provides the means to achieve this goal. The chosen basis has the property that each wavelet coefficient corresponds to a function value at a grid point. This is a key element of the method, since we have a fast algorithm to move between the physical space, where the computations are made and the wavelet space, where

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we choose the point to include in our SPR by thresholding the wavelet coefficients.

Modeling of non-linear semiconductor devices e.g. transistors or diodes produces functions (carrier concentrations and potential distribution) that are smooth almost everywhere in the domain except in small regions of sharp variations near p-n junctions. We would like to use a nonuniform grid, which is fine around sharp variation locations and coarse in areas where the solution is smooth. Application of the SPR may give us an opportunity to consider our grid as a dynamic object which is fully integrated into the solution. A nonuniform grid becomes fully adaptive. Changes in the grid follow the changes in the solution on each time step.

STATEMENT OF THE PROBLEM

The basic physical model consists of three coupled partial differential equations: Poisson equation for the electric field and two continuity equations for electrons and holes. They are supplemented by the expressions for the electron and hole current densities. Since the goal of this paper is to demonstrate the wavelet's potential and to simplify the analysis, the drift-diffusion approximation is used in the current density expressions. The mathematical model consists of Poisson's equation

$$\nabla^2 U = -\frac{q}{\epsilon}(N_d - N_a + p - n) \tag{1}$$

The electron and hole carrier concentrations are found from the continuity equations

$$\frac{\partial n}{\partial t} - \frac{1}{q}\nabla \cdot \mathbf{J}_n = 0 \tag{2}$$

$$\frac{\partial p}{\partial t} + \frac{1}{q}\nabla \cdot \mathbf{J}_p = 0 \tag{3}$$

and the electron and hole current densities

$$\mathbf{J}_n = q\mu_n(\mathbf{E})n\mathbf{E} + qD_n\nabla n \tag{4}$$

$$\mathbf{J}_p = q\mu_p(\mathbf{E})p\mathbf{E} - qD_p\nabla p \tag{5}$$

It remains to specify boundary conditions for a particular geometry. Figure 1 shows a representative example of a two dimensional crossection of a silicon abrupt diode. The potential, and electron and hole carrier concentrations satisfy appropriate initial, boundary and interface conditions. In general, there are semiconductor/conductor interfaces (contacts), semiconductor/isolator interfaces and outside boundaries.

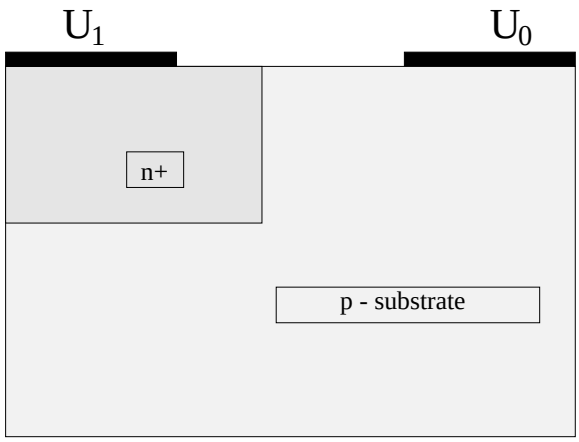

Fig. 1. Idealized crossection of a silicon abrupt diode.

In order to solve the drift-diffusion equations, it is convenient to express all variables (the potential, the electron and hole densities, current densities, electron and hole mobilities) in terms of scaled quantities.

The spatial discretization is done by a scheme of Scharfetter-Gummel type.

TRANSIENT SOLUTION

In scaled form, the basic semiconductor equations can be written as

$$\begin{aligned} g_1(\mathbf{U}, \mathbf{n}, \mathbf{p}) &= 0 \\ g_2(\mathbf{U}, \mathbf{n}, \mathbf{p}) &= 0 \\ g_3(\mathbf{U}, \mathbf{n}, \mathbf{p}) &= 0 \end{aligned} \tag{6}$$

where vectors $\mathbf{U}, \mathbf{n}, \mathbf{p}$ are now the SPR of the normalized electrostatic potential and carrier concentrations.

In our calculations we will follow the modified Gummel iterations The resulting system of ODE can be solved by the ODE solver. We use the fourth order Runge-Kutta method for the solution.

The iterative procedure to solve the problem (6), can be presented in seven steps:

1. Set initial values for the function p and n and fix a threshold value δ.
2. Obtain SPR for p, n and U.
3. Solve the FD approximation of the Poisson's equation g_1 and obtain SPR for the potential.
4. Make one step in the continuity equation g_2 for n using Runge-Kutta method and using the potential from the previous step.
5. Repeat previous step in the continuity equation g_3 for p.
6. Update all SPRs.
7. Goto 2.

The basic difficulty in the solution of the transient system is the requirement that the numerical method must be unconditionally stable.

NUMERICAL RESULTS

Example 1. Consider a silicon $p - n$ junction in Figure 2. The volume concentration of the implanted

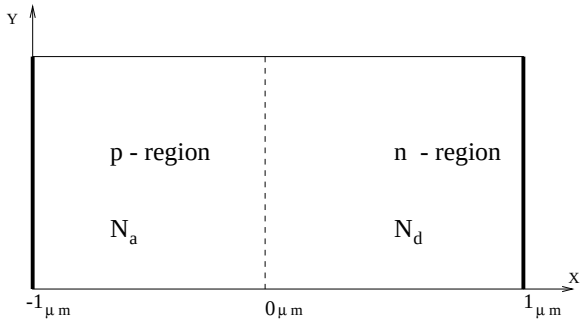

Fig. 2. Silicon $p - n$ junction.

acceptors $N_a = 5 \times 10^{15} cm^{-3}$, the volume concentration of the implanted donors $N_b = 1 \times 10^{15} cm^{-3}$. The resulting electron and hole concentrations for an abrupt silicon $p - n$ junction with zero external bias are presented in the Figure 3. The potential distri-

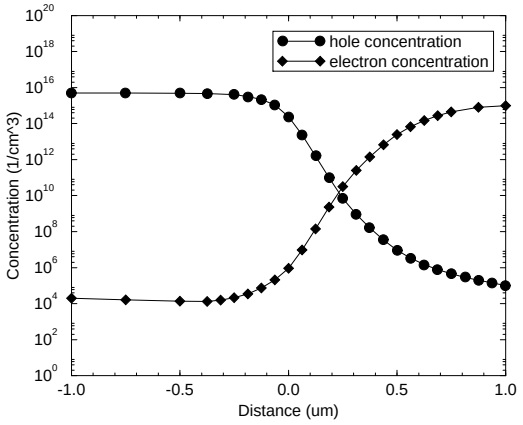

Fig. 3. Electron and hole carrier concentrations in a silicon $p - n$ junction.

bution is shown in Figure 4. Markers on the curves show corresponding mesh points. We can see from the figures that all components of the solution have their own meshes. In case of the potential this mesh is quite coarse. Numerical calculations show that we need several hundred iterations (from 250 to 400) to get a steady state solution of the equations. The number of iterations depends on the value of the thresh-

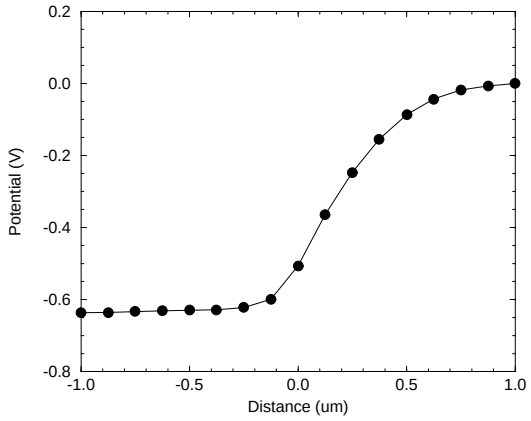

Fig. 4. The potential distribution for an abrupt silicon $p - n$ junction at zero external bias.

old parameter δ. As we have mentioned before, the smaller parameter leads us to the finer mesh and more iteration steps to reach the steady state.

Example 2 Consider an abrupt $n^+ - p$ diode in 2D as you can see on the Figure 1. The structure is of $2\mu m \times 2\mu m$ with contacts of length $0.5\mu m$ on the upper left and right of the device. The doping concentration under the left contact $N_d = 1.482 \times 10^{16}\ cm^{-3}$ (n-type region has a square shape of $1\mu m \times 1\mu$). In the substrate we assume $N_a = 1.482 \times 10^{15}\ cm^{-3}$ (p-type). Figures 5 and 6 show us the distribution of the electron concentration and the corresponding mesh. The number of nodes in the mesh is 414. Figures 7 and 8 show us the distribution of the hole concentration and corresponding mesh. The number of nodes in the mesh is 327. The number of iterations is about 1000. The above meshes have been adapted with the threshold parameter $\delta = 10.0\%$. In contrast, the full mesh of the uniform grid consists of 1089 nodes. The computations are compared to simulations results obtained with the full mesh in the refinement limit. The number of iterations in numerical experiments was in the range of 1000-5000 for different values of the threshold parameter δ.

Simulation results were compared also with the results from the ATLAS simulator. They are comparable, though not exactly identical, due to the slightly different physical parameters, like the intrinsic density, mobilities.

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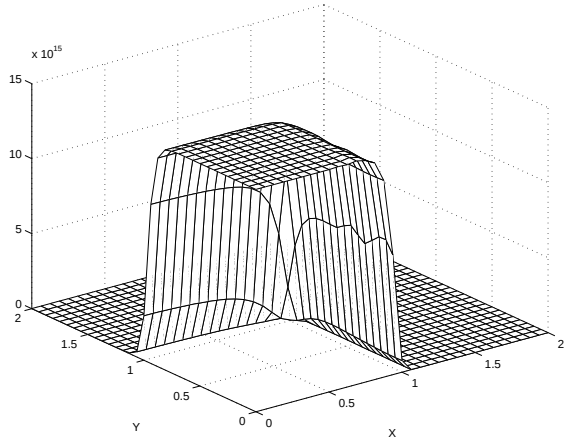

Fig. 5. The electron concentration for a 2D abrupt diode with zero external bias.

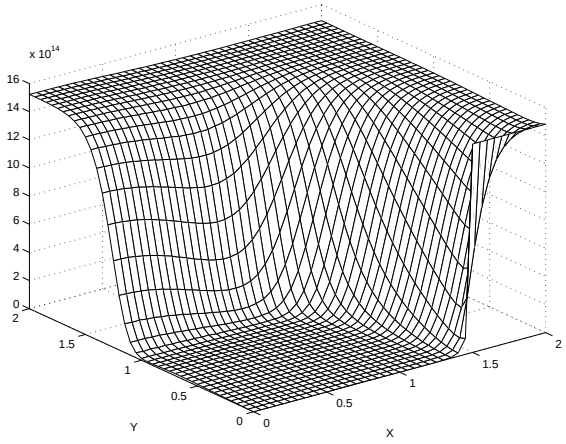

Fig. 7. The hole concentration for a 2D abrupt diode with zero external bias.

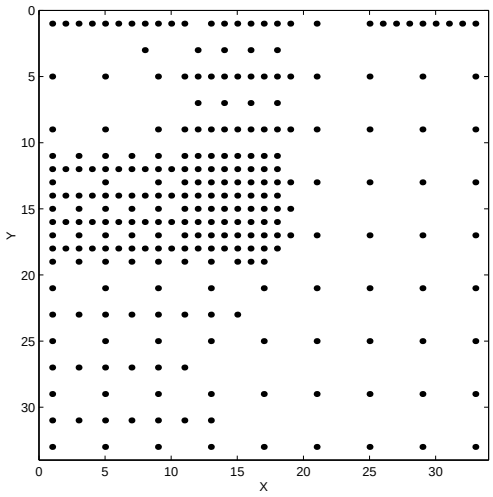

Fig. 6. Grid points of the electron concentration for a 2D abrupt diode with zero external bias.

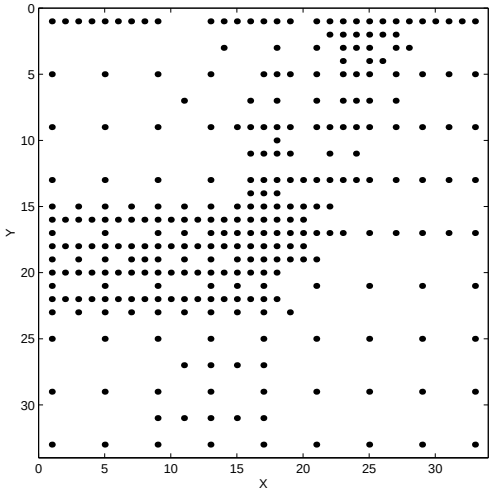

Fig. 8. Grid points of the hole concentration for a 2D abrupt diode with zero external bias.

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